Trigonometry

Trigonometry

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If you have any questions, comments or suggestions about this material, contact Mike Weimerskirch at weim0024@umn.edu

Contents

Introduction

This book covers the major topics within the study of trigonometry, including vectors and their applications. At the University of Minnesota, this material is 75% of the *PreCalculus II* course, with the remaining 25% of that course covering algebraic topics which are included in a separate text. It comprises approximately 10-12 weeks worth of material at the college level; a typical college student would spend about 120 hours total learning this material.

These materials are used as part of an active learning course, and the interplay between in-class activities that allow students to explore and discover ideas and a more formulaic study of the necessary computations is important. Each chapter has suggestions for active learning problem-solving activities designed to develop higher-order thinking skills. The video lessons in the chapter sections address a second objective, the skill-building component of mathematics. Can you successfully carry out the calculations necessary to find answers to a variety of applications? Conceptual understanding of the material is important, and instructors are encouraged make use of activities that help students advance the computational skills developed in the video lessons.

This video textbook project has been funded by the University of Minnesota School of Mathematics and by grants from the University of Minnesota Libraries Partnership for Affordable Content. My thanks go out to the many people who have contributed to this project including Susan Tade, Jennifer Englund, Colin Marron, Hue Yang, Andrew Matthews, James Ondrey and Melissa Olson with the Unversity of Minnesota's Academic Technology Support Services for their assistance with the recording, animation and post-production of the videos, Robbie Hank, Shelley Kandola and other instructors who helped with the design of the Beamer slideshows that form the basis of the visual presentation, Stan Pride, Kevin Charles and others who have helped with other aspects of video editing to enhance the student experience, David Ernst for being the catalyst of the entire project by developing the *Open Textbook Network* and Kristi Jensen, Shane Nackerud and others with the University of Minnesota Libraries who have made this vision come to life in the final production stages.

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Chapter 1 - Unit Circle and Definitions

Chapter 1 begins with the geometric (right triangle) definition of sine, cosine and tangent. It then develops the unit circle and the trigonometric definitions of sine, cosine and tangent. It does not discuss the connection between the geometric and unit circle definitions, as that is more appropriate for a formal geometry course. The chapter ends with other properties of trig functions, including the reciprocal functions.

Activities - Chapter 1

Note to Students:

Students should be aware that these worksheets are not `fill in the blank' worksheets. **We have intentionally given you no space to write answers on the worksheets.** You should have a notebook for your work, perhaps even beginning by sketching ideas on a white board or scratch paper until you have sufficient organization to put a coherent explanation in your notebook. Note also that the expectation is to write `explanations' and not `answers'. These activities are not designed to teach computational skills, but instead are designed to introduce mathematical concepts. The process used to solve problems is the focus, not the end result.

Activity 1a – Special Triangles

This first activity on the 45°- 45°- 90° triangle and the 30°- 60°- 90° triangle is designed to develop the building blocks to complete the Unit Circle in Topic 1.4. The Unit Circle is introduced in Topic 1.2 and this material in important to build upon the definitions given in Topic 1.2 to develop the complete Unit Circle in Topic 1.4. It can be done immediately prior to Topic 1.4, or can be done earlier in Unit 1, it relies only on some ideas from geometry.

Within the activity are instances where you will want to be familiar with simplifying square roots (or as it is sometimes called, simplifying `radicals'). You may wish to remind yourself of these simplification rules prior to this activity.

Special Triangles

Activity 1b – Piston Motion

This activity only requires knowledge of the unit circle (Topic 1.4) and therefore can be done in Unit 1, although it requires a fair amount of mathematical sophistication. This activity will help build problem-solving skills, and asks a deep question that will require a fair amount of time. It is a stand-alone activity, and could be done just about anytime during a Trig course. Ideas from Topic 1.4 are important, as well as the Pythagorean Theorem.

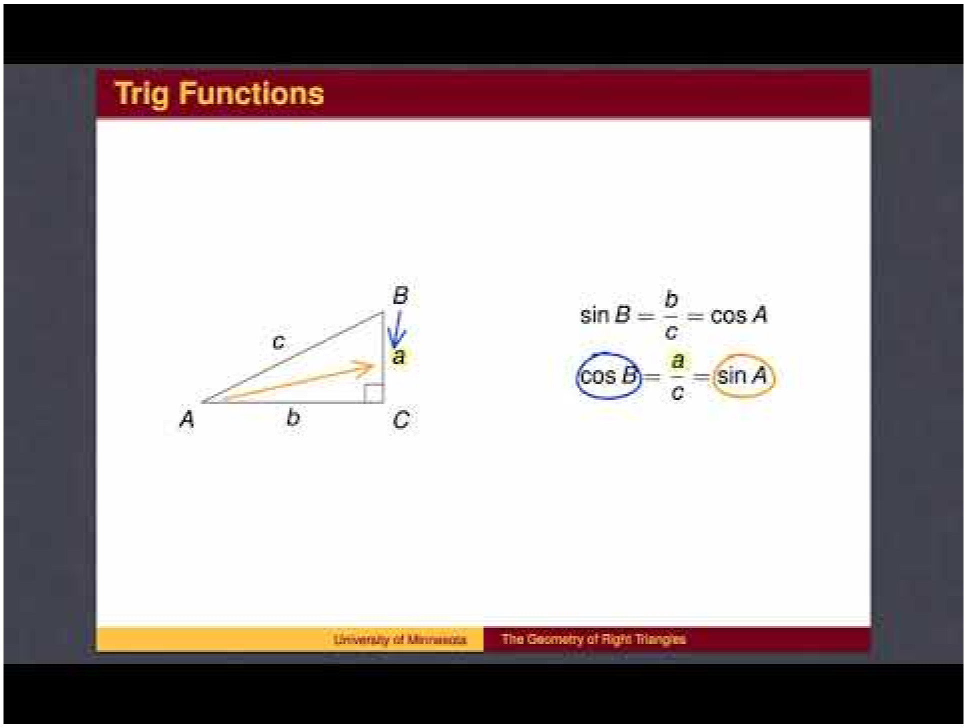
It is best to see the piston motion in action, and the worksheet provides a link to desmos.com with an animation.

Piston Motion

Topic 1.1
The Geometry of Right Triangles

The Geometry of Right Triangles begins the study of Trigonometry. It begins with the geometric definitions of the three main trig functions. Some instructors prefer to begin with the unit circle approach and then proceed to the right triangle approach. We chose to put this lesson before *Unit Circle – Part I and Part II*, though it could be done afterward. It gives a conceptual introduction to the inverse trig functions, enough to be able to solve right triangles and the focus here is on the solving process. A fuller, more formal treatment of inverse trig functions is given in *Inverse Sine and Inverse Cosine Functions*, with further details of the domain and range of the inverse trig functions given in *Domain and Range of Trig and Inverse Trig Functions*.

It is not the intent of this lesson to discuss the similarity of triangles and the fact that the geometric definition is well-defined.



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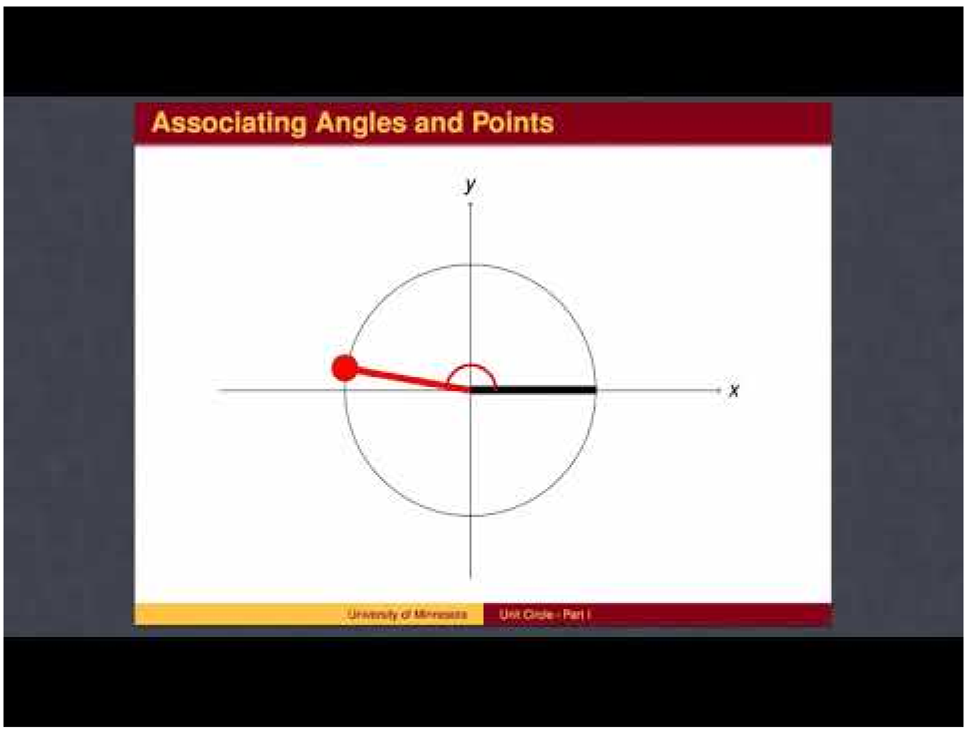
Topic 1.2
The Unit Circle - Part I

The *Unit Circle – Part I* establishes the connection between angles measured counterclockwise from the positive side of the x-axis and points on a circle of radius 1 and formalizes this as a function from angles to points (ordered pairs). From this, the *sin* and *cos* functions are defined. Value of sin and cos are found for angles which are multiples of 90°. Values which come from the 30° – 60° – 90° triangle and the 45° – 45° – 90° triangle are presented in the *Unit Circle – Part II* lesson.



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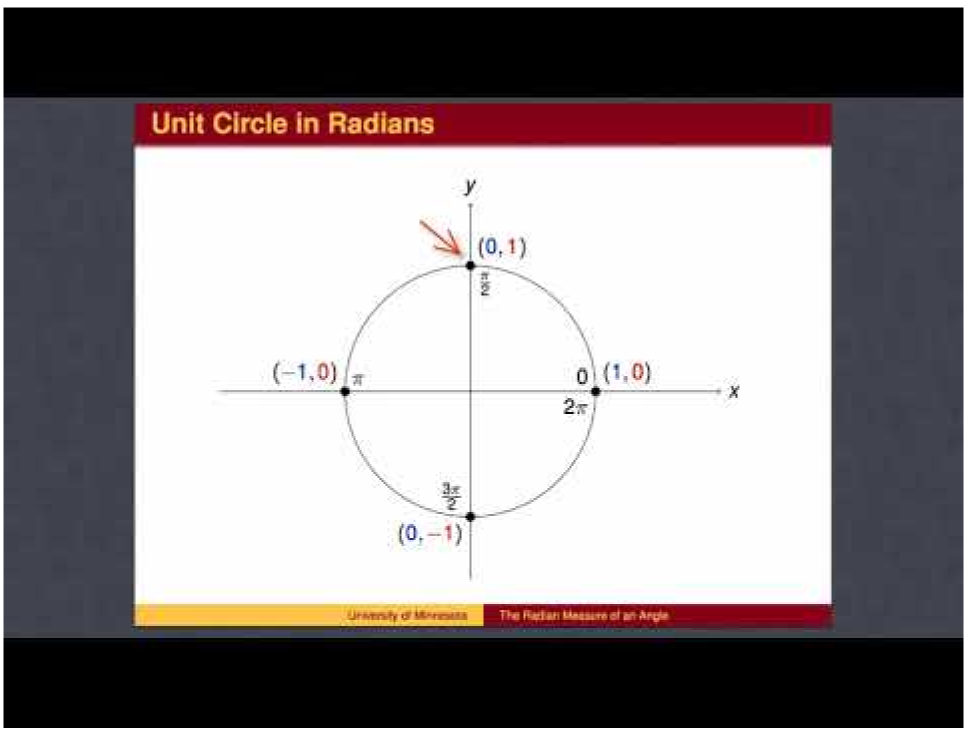
Topic 1.3
The Radian Measure of an Angle

The Radian Measure of an Angle establishes the connection between angles measured on the unit circle in degrees and the arc length around the circumference of the circle. Sine and cosine values are referenced, though they are not taught in this lesson, nor are they important here. Conversions from radians to degrees and from degrees to radians are performed.



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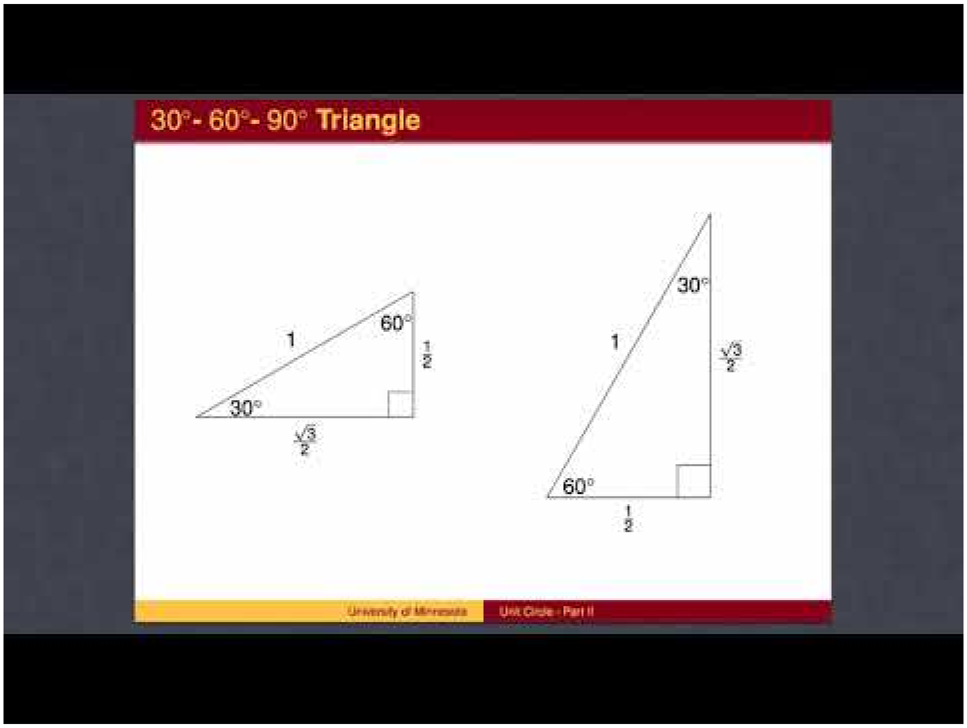
Topic 1.4
The Unit Circle - Part II

The *Unit Circle – Part II* finishes the work begun in the *Unit Circle – Part I* by discussing the 30° – 60° – 90° triangle and the 45° – 45° – 90° triangle. It includes both degree and radian measure. By the end of this lesson, students should be able to find sine and cosine values for 16 angles between 0° and 360°.



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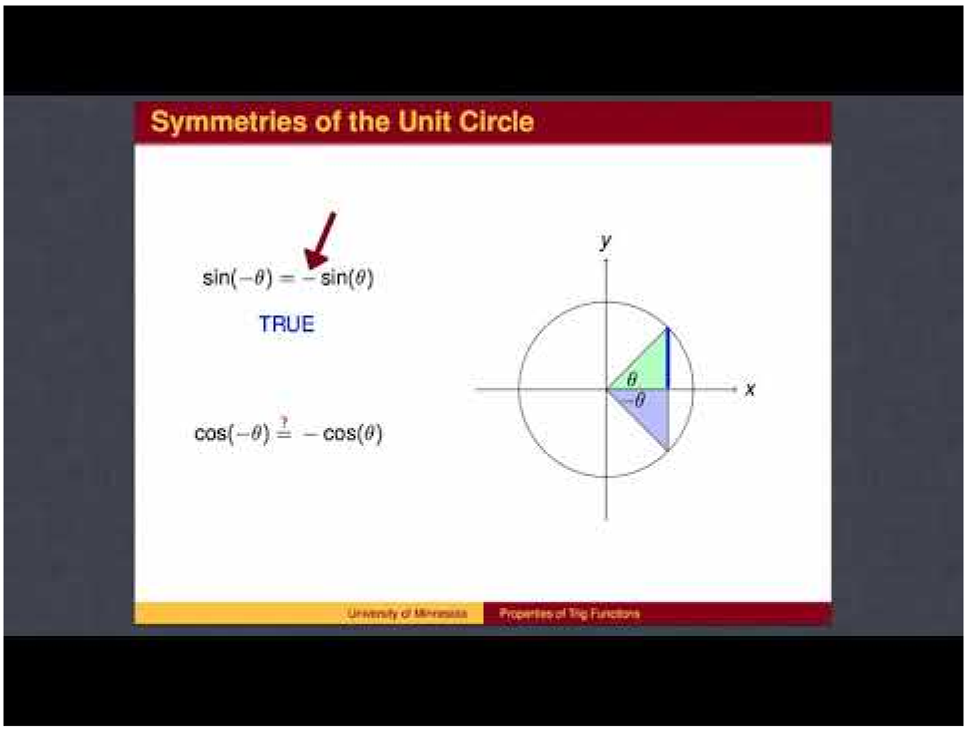
Topic 1.5
Properties of Trig Functions

Properties of Trig Functions expands the special unit circle values from 0 to 2π to include angles larger than 2π and negative angles. By the end of the lesson, students should be able to find the sine and cosine values for arbitrarily large angles. Information about the quadrant an angle is in should allow students to find these values having only memorized the unit circle from 0° to 90°.



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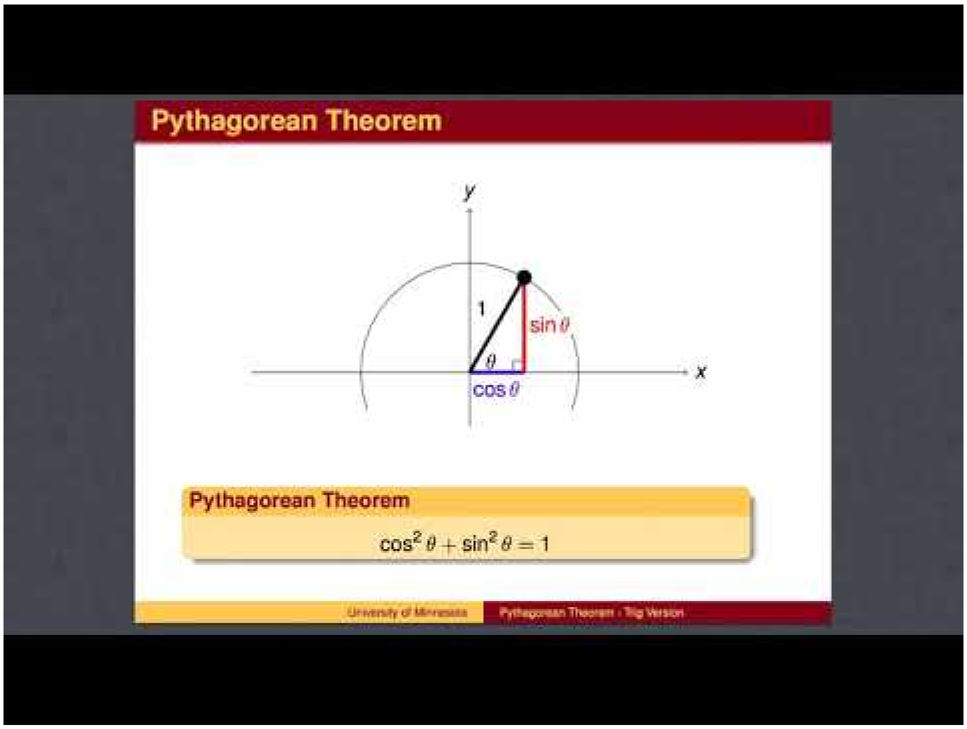
Topic 1.6
Pythagorean Theorem - Trig Version

Pythagorean Theorem – Trig Version allows students to find the value of sin when given cos and vice versa. Examples are done using the standard trigonometric method with the hypotenuse of length 1, and also using the geometric definition where the hypotenuse can have arbitrary length.



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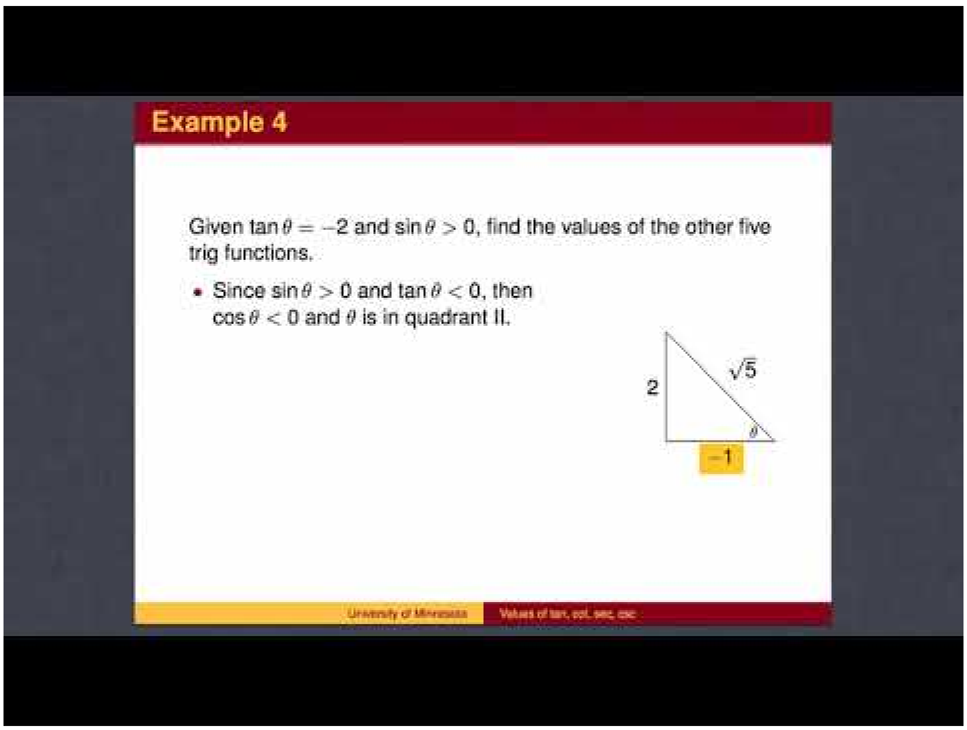
Topic 1.7
Finding Values of tan, cot, sec, csc

Finding Values of tan, cot, sec, csc defines the four functions *tan*, *sec*, *cot* and *csc*. Given the value of any one of the six trig functions, the values of the other trig functions can be found and several examples are given.



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Chapter 2 - Graphing

Chapter 2 is devoted to the graphs of trig functions. Most of the attention is spent on sine and cosine waves, and understanding amplitude, frequency, wavelength and phase shift. The graphs of the other four trig functions are discussed briefly.

Activities - Chapter 2

Activity 2a – Transformations of Functions and their Graphs

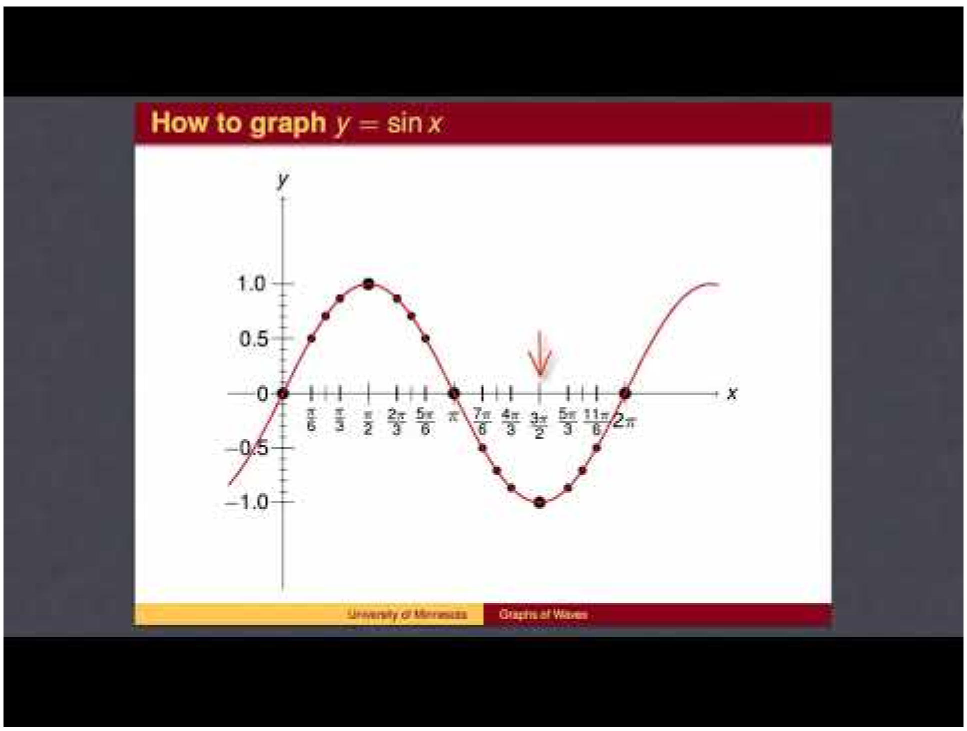
This activity prepares students for graphing sine and cosine waves. It should help students understand Topic 2.2 (Amplitude), Topic 2.3 (Frequency, Wavelength and Period) and Topic 2.5 (Phase Shift). It does not depend on any knowledge of the graphs of $y = \sin\ x$, or $y = \cos\ x$, and therefore is a good way to introduce the unit on graphing trig functions. The activity does rely on functions from algebra, and may also serve as a review of these functions.

A student preparing for Calculus should at some point be familiar with the graphs of a Catalog of Common Functions, such as lines ($y = mx + b$), parabolas ($y = x^2$) and other power functions ($y = x^3, x^4, x^5 \ldots$), the reciprocal function ($y = 1/x$), exponential functions $(y = Ke^{rt}, y = Ke^{-rt}$) and logarithmic functions ($y = \ln\ x, y = \log\ x$), as well as the trig functions. The focus here is that once you have learned a basic graph, like $y = \sin\ x$, you will then be able to **shift**, **stretch** and **flip** that graph to find new graphs.

General Graphing

Topic 2.1
Graphs of Waves

Graphs of Waves shows the details of the graphs of $y = \sin x$ and $y = \cos x$. Close attention is paid to graphing one complete wave from 0 to 2π, including points corresponding to each of the special unit circle angles. This is far more detail than is necessary for most applications, but does show the roundedness of the waves. The pattern then repeats and this periodic nature is shown briefly to give students an understanding of the long range behavior of the functions.



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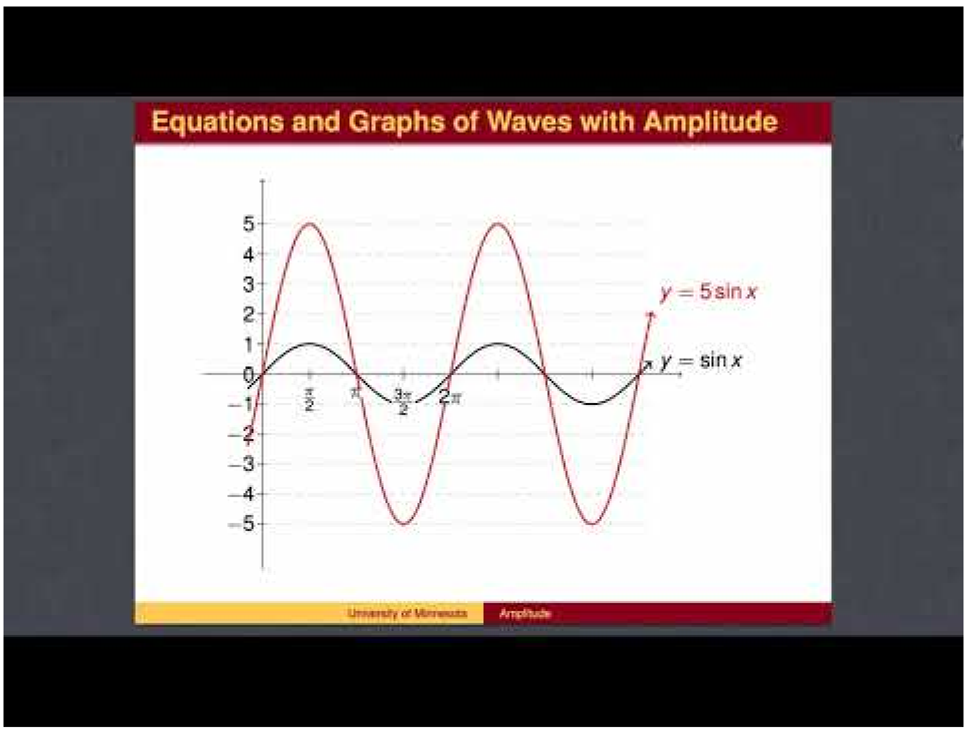
Topic 2.2
Amplitude

Amplitude is the first of three lessons building toward the general wave function. Future lessons deal with *Frequency, Wavelength and Period* and *Phase Shift*. By the end of this lesson, students should be able to write an equation from the graph, and should be able to graph the wave, given its equation.



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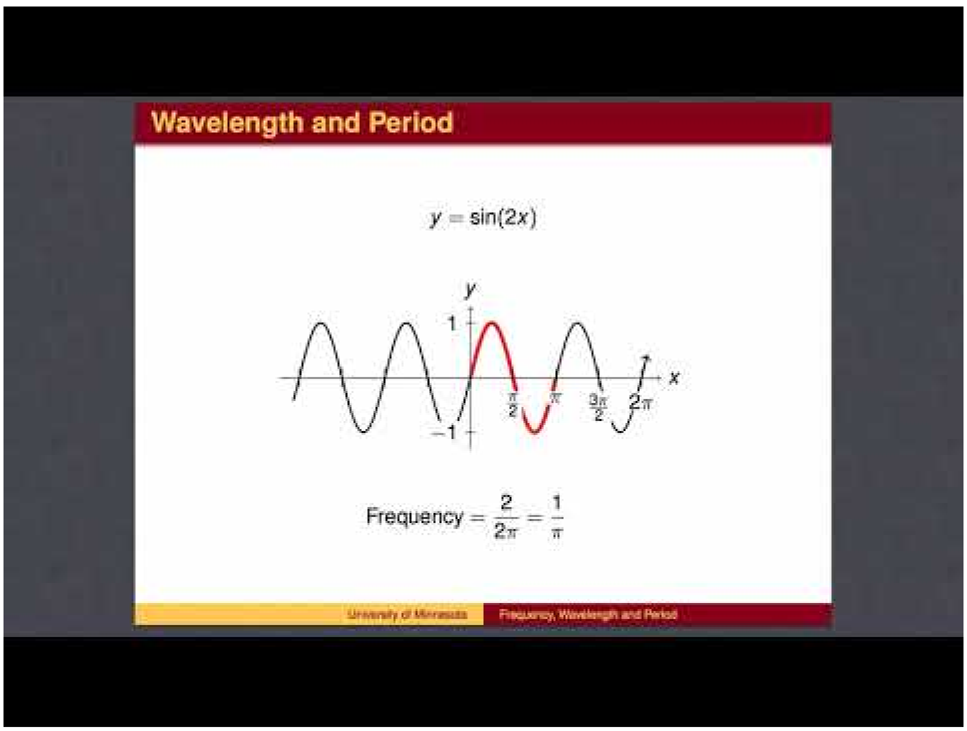
Topic 2.3
Frequency, Wavelength and Period

Frequency, Wavelength and Period extends the techniques for graphing waves begun in the lesson on *Amplitude*. The physical concepts of wavelength and frequency are defined here, but are not stressed. The term period is used in place of wavelength in most examples. The wave model $y = A\ sin(Bx)$ is established in this lesson. A final lesson on *Phase Shift* realizes the full generalization $y = A\ sin(Bx + C) + D$.



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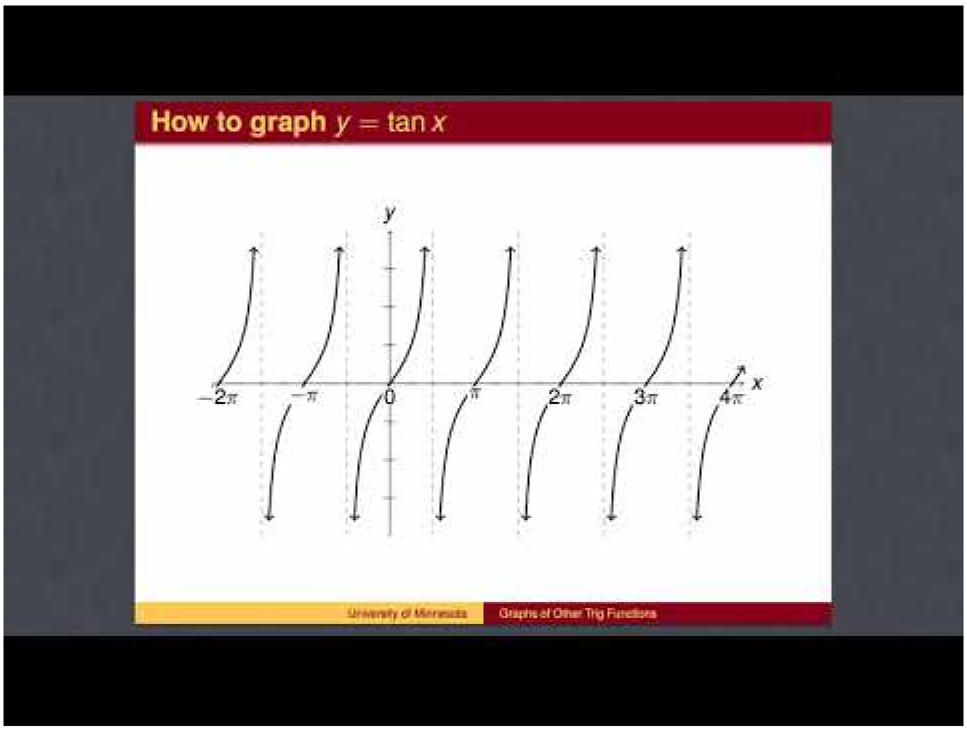
Topic 2.4
Graphs of Other Trig Functions

Graphs of Other Trig Functions shows the graphs of *y* = *tan x*, *y* = *cot x*, *y* = *sec x* and *y* = *csc x*. The lesson does not cover transformations of these graphs as they have little application. The main use of the graph of *y* = *tan x* is to be able to talk about *y* = *arctan x*, which is useful in some contexts.



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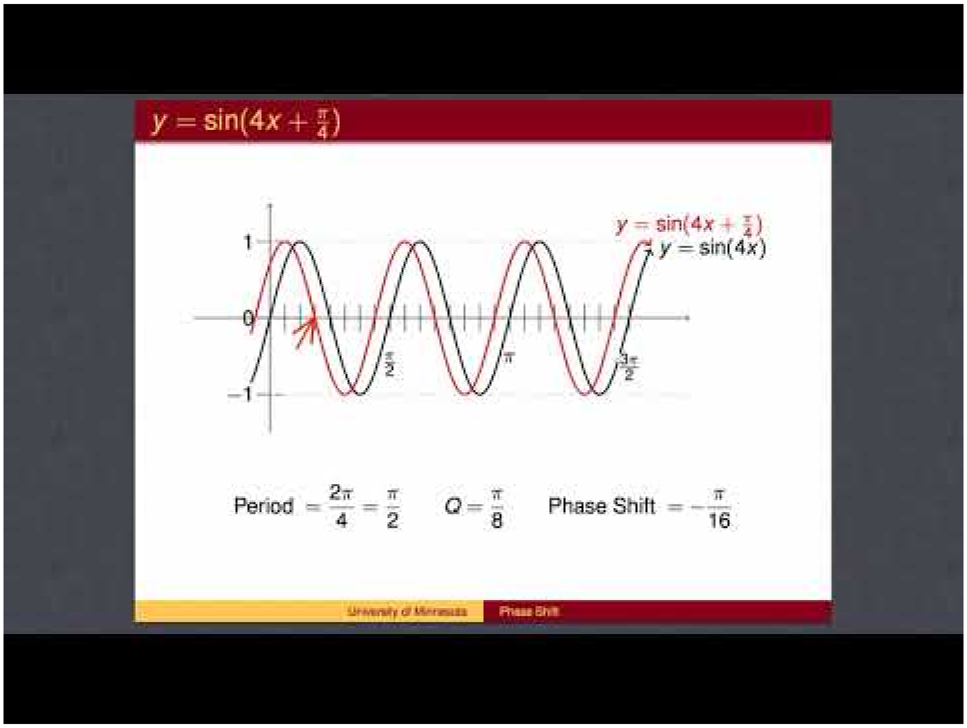
Topic 2.5
Phase Shift

Phase Shift is the third and final lesson in establishing the general wave model

$$y = A \sin(Bx + C) + D$$

It relies on earlier lessons on *Amplitude* and *Frequency, Wavelength and Period*.



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Chapter 3 - Trig Formulas and Inverse Functions

Chapter 3 begins with the inverse functions *arcsin* and *arccos* and a discussion of domain and range. The middle sections are devoted to many of the trig identities, like double and half angle formulas. The last two sections deal with solving trig equations.

Activities - Chapter 3

Activity 3a – Inverse Functions

This activity explores the details of finding an inverse to a function and the implications on the domain and range of the function and its inverse. It is related to the discussion of the inverse trig functions in Topics 3.1, 3.2 and 3.3.

It is a very visual lesson, and students should have some piece of technology to graph several functions at once.

Inverse Functions

Activity 3b – Angle Sum Formulas Part I

This activity is the first of three that generates formulas useful to simplify trig expressions. These formulas are used in Topics 3.5, 3.6 and 3.7.

There are several issues that students will have to sort out, including interpreting function notation, some geometry, symmetry, and some pretty intense algebra.

Angle Sum Worksheet – Part I

Activity 3c – Angle Sum Formulas Part II

This activity builds upon Activity 3b and continues to generate formulas used in Topics 3.5, 3.6 and 3.7.

Again, students will have to sort out function notation, geometry, symmetry, and algebra.

Angle Sum Worksheet – Part II

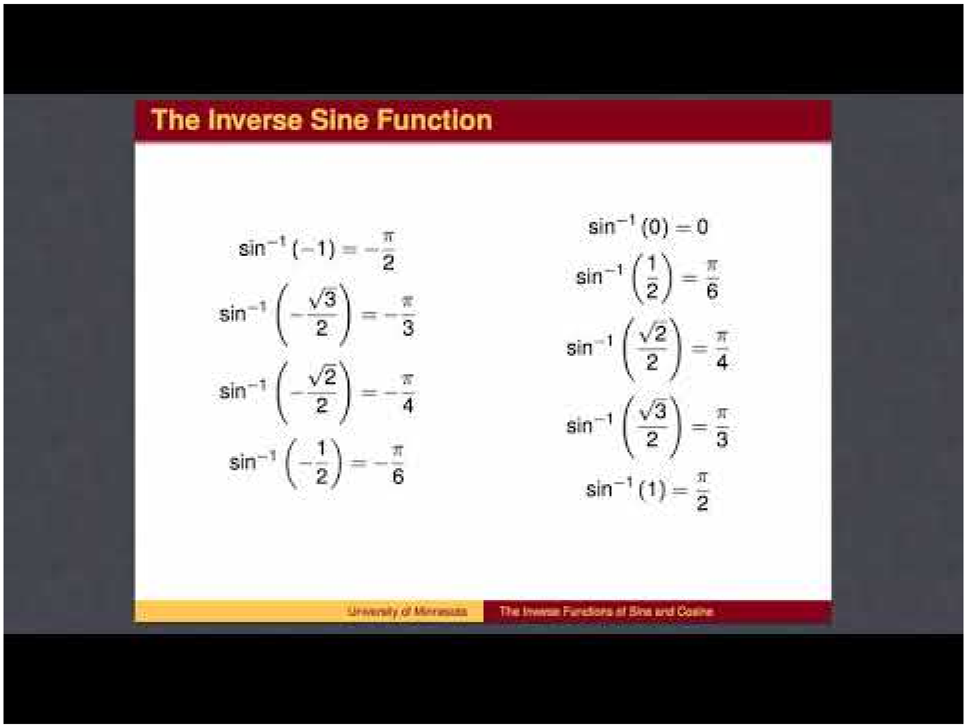
Activity 3d – Double Angle Formulas

This activity is directly related to the formulas in Topic 3.7. It finishes the work that began in Activity 3b and 3c.

Double Angle Formulas

Topic 3.1
The Inverse Functions of Sine and Cosine

The Inverse Functions of Sine and Cosine introduces the inverses to the basic trig functions. These functions $y = \sin^{-1}(x)$ and $y = \cos^{-1}(x)$ are also denoted $y = \arcsin(x)$ and $y = \arccos(x)$ respectively, as is mentioned in *Right Triangle Geometry*. It relies on the unit circle values found in *The Unit Circle – Part I* and *Part II*. It is helpful to have an understanding of how the graph of a function illustrates the domain and range of a function, and further details of the domain and range of the inverse trig functions are given in *Domain and Range of Trig and Inverse Trig Functions.*



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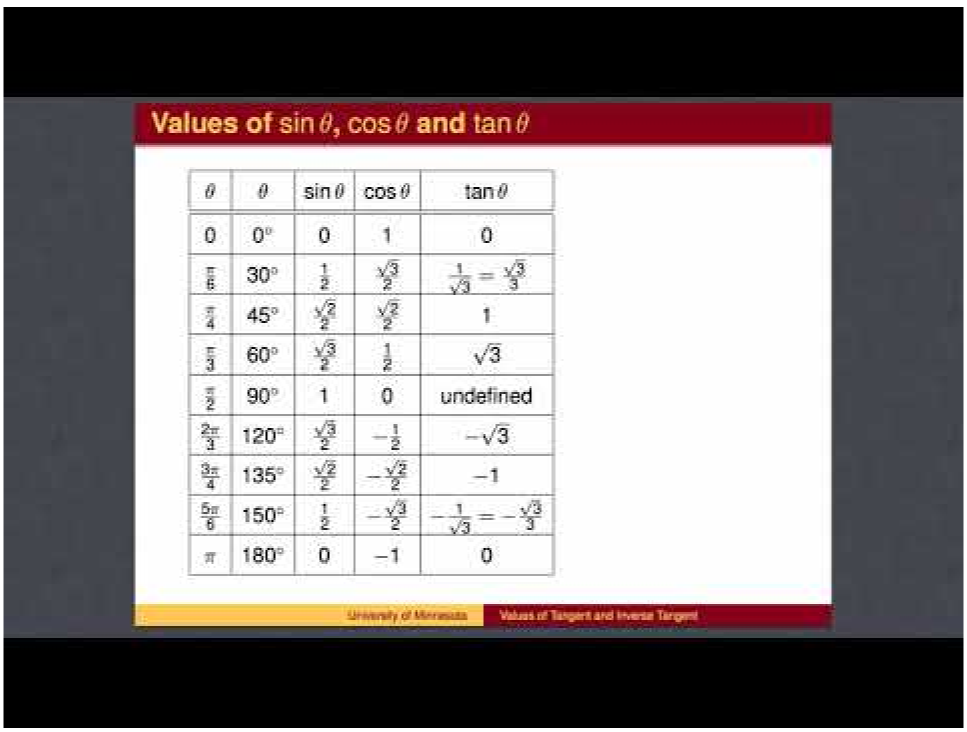
Topic 3.2
Tangent and Inverse Tangent

Values of Tangent and Inverse Tangent develops values of the tangent function for unit circle angles and the inverse tangent function. It is assumed that students are familiar with $y = \sin^{-1}(x)$ and $y = \cos^{-1}(x)$ which are found in *The Inverse Functions of Sine and Cosine*. It is also assumed that students are well-versed in handling the simplification of fractions within fractions involving radicals, for example, being able to simplify $\dfrac{1/2}{\sqrt{3}/2}$. We are neutral on whether $\dfrac{\sqrt{3}}{3}$ or $\dfrac{1}{\sqrt{3}}$ is the preferred form for $\tan(\dfrac{\pi}{6})$ and present both. We intentionally chose not to address inverses for $y = \cot(x)$, $y = \sec(x)$, and $y = \csc(x)$ since these inverse functions are not widely used.



θ	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0°	0	1	0
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	undefined
$\frac{2\pi}{3}$	120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
$\frac{3\pi}{4}$	135°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
$\frac{5\pi}{6}$	150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
π	180°	0	-1	0

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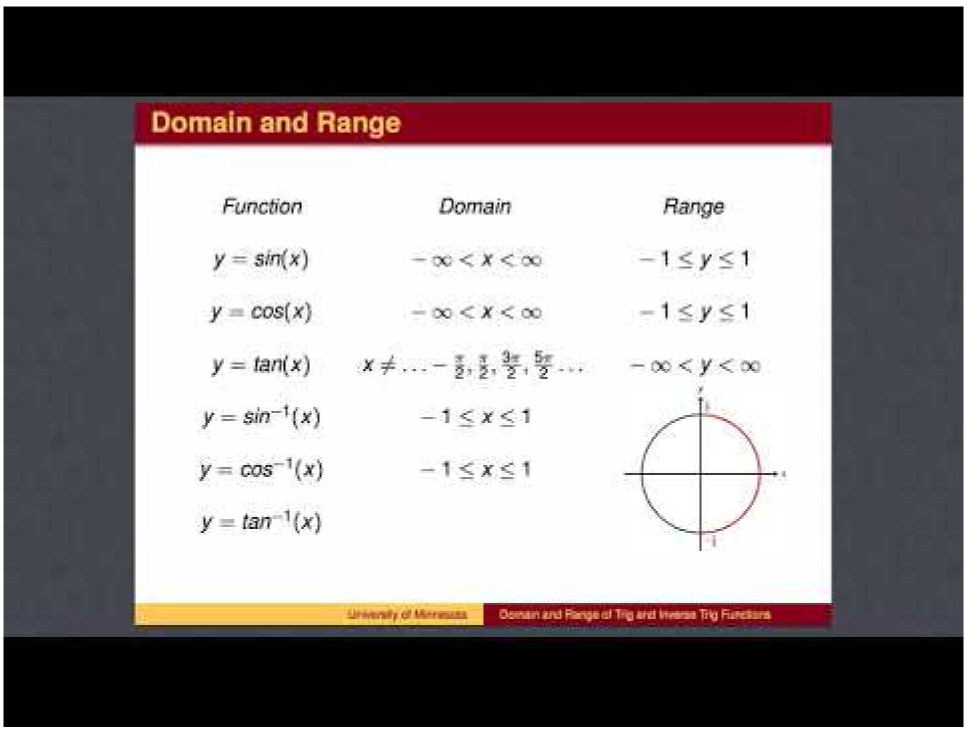
Topic 3.3
Domain and Range of Trig and Inverse Trig Functions

Domain and Range of Trig and Inverse Trig Functions covers the specifics of the domain and range of $y = \sin(x)$, $y = \cos(x)$, and $y = \tan(x)$ and their inverses. It is assumed that students are familiar with these functions and can find values for the unit circle. Background material can be found in *The Inverse Functions of Sine and Cosine* and *Values of Tangent and Inverse Tangent*.



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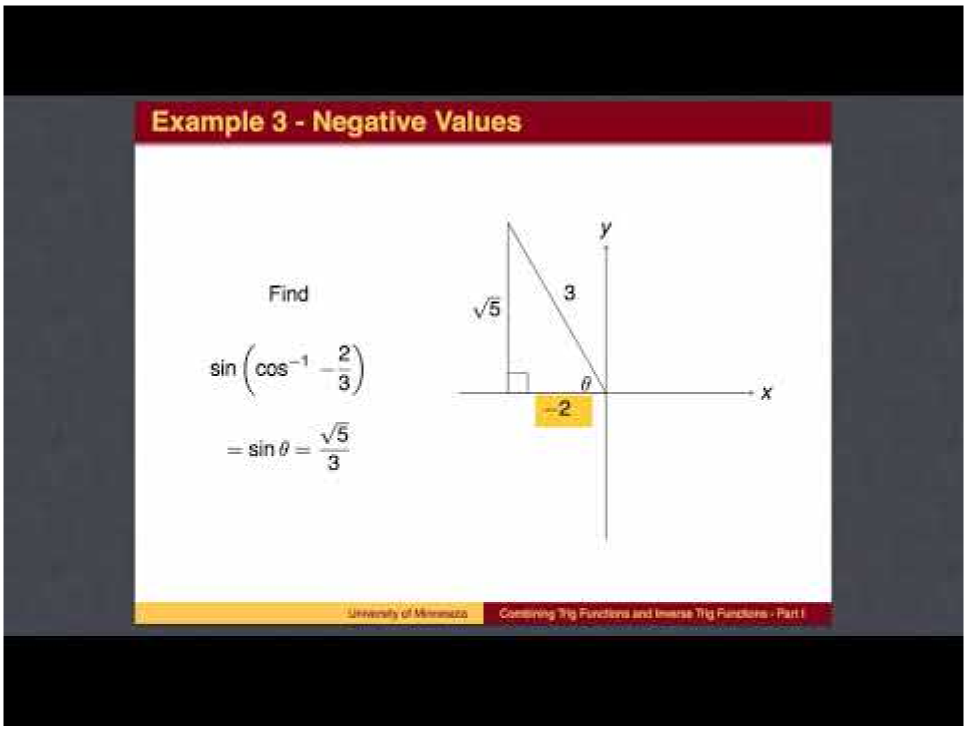
Topic 3.4
Combining Trig Functions and Inverse Trig Functions - Part I

Combining Trig and Inverse Trig Functions – Part I covers several examples of how these functions can be combined. The emphasis is on developing the notation and understanding at each step whether the object in question is an **angle** or a **number**. Attention is given to the domain and range of the functions. These issues are important for lessons in solving triangles using *The Law of Sines* and *The Law of Cosines*. Example 2 in the video is a repeat from *Pythagorean Theorem – Trig Version*.



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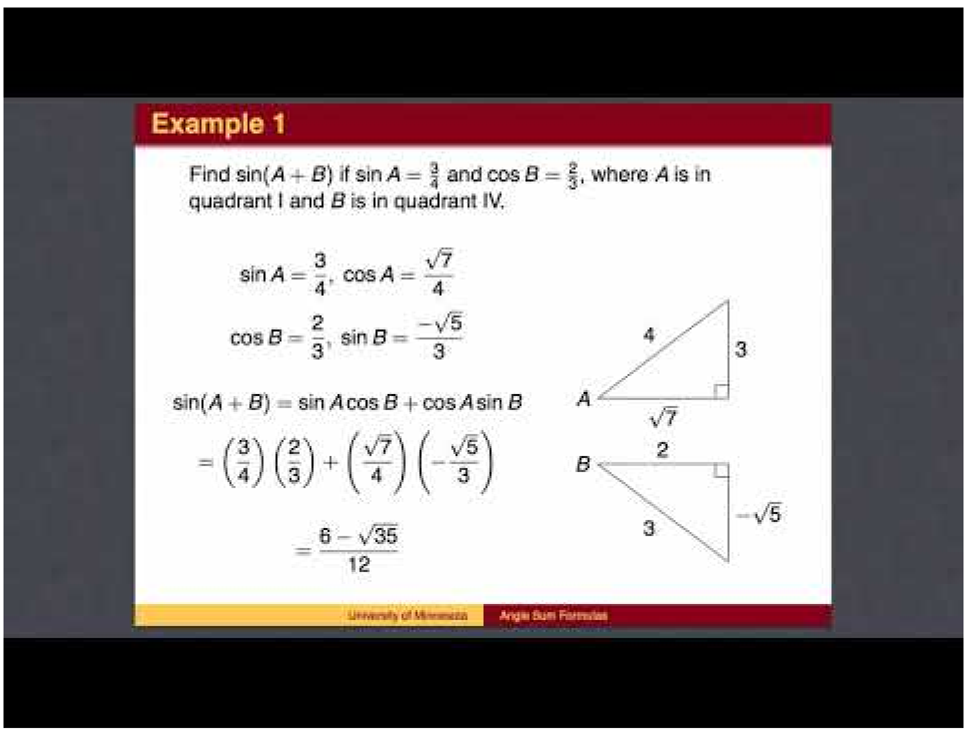
Topic 3.5
Angle Sum Formulas

Angle Sum Formulas covers the calculation aspects of working with the angle sum formulas. It is assumed the students already know each formula, as no attempt to derive them is made here. They are most commonly proved by geometric means, but we prefer a more algebraic approach which can be found in the corresponding in-class activity.



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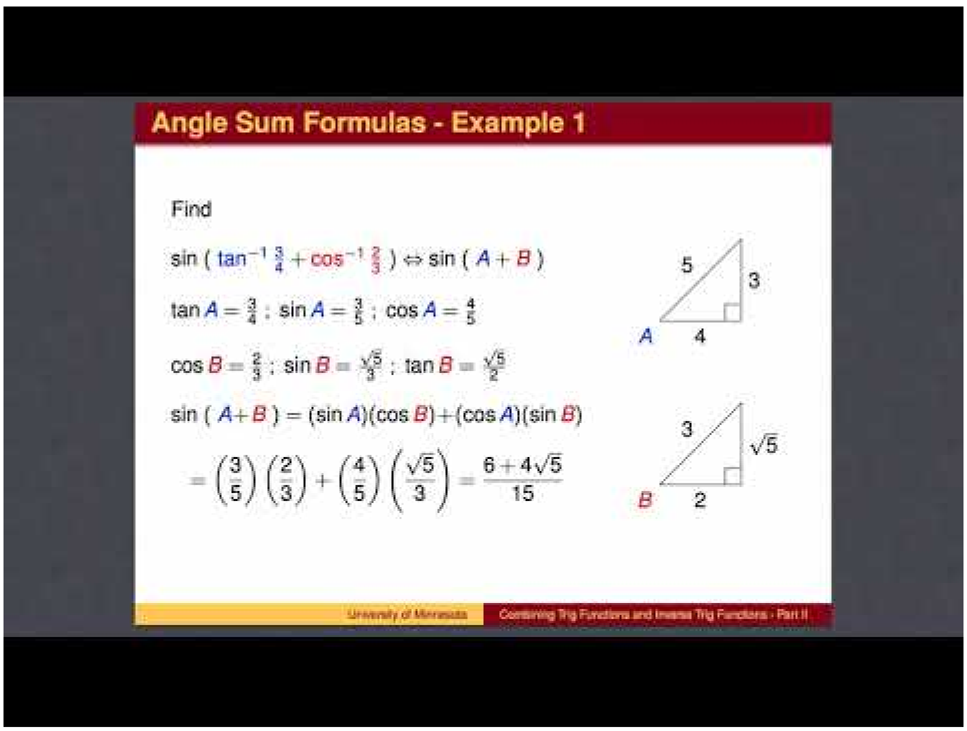
Topic 3.6
Combining Trig Functions and Inverse Trig Functions - Part II

Combining Trig and Inverse Trig Functions – Part II covers several examples of how these functions can be combined. The emphasis is on the selection of the appropriate formula and finding the necessary values to input. Details of the simplification are not dealt with in the video. Students will need practice, or otherwise be adept at handling the simplification of complicated fractional expressions with radicals.



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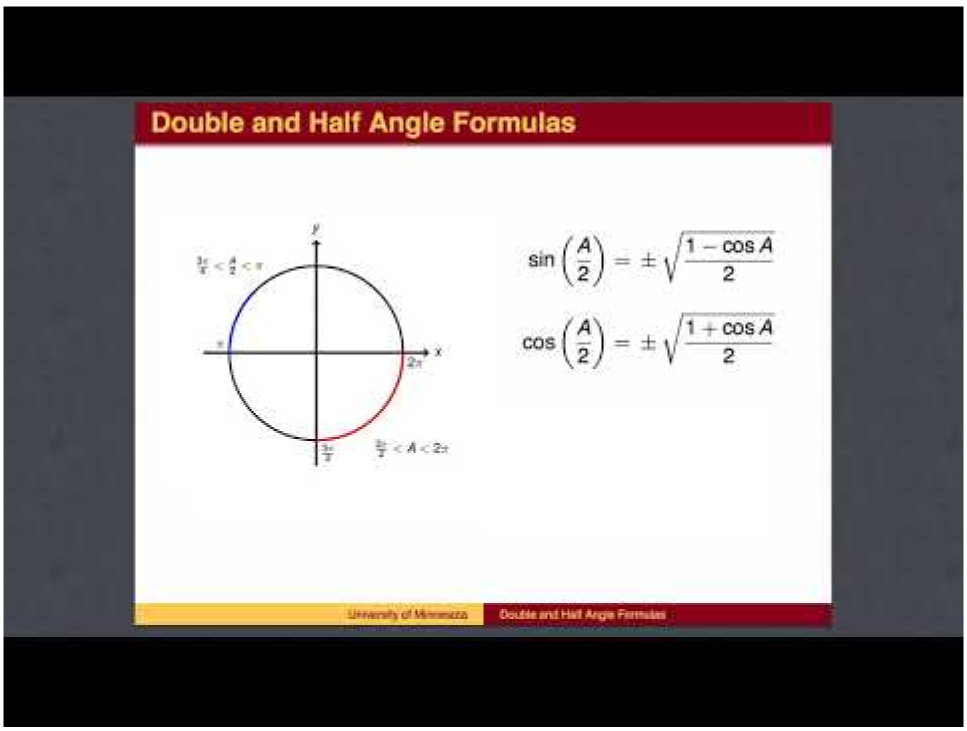
Topic 3.7
Double and Half Angle Formulas

Double and Half Angle Formulas covers examples similar to *Combining Trig and Inverse Trig Functions, Parts I-II*. Students should be able to derive the formulas on their own, or otherwise be presented with the formulas. The emphasis is on calculating numerical answers, and on the selection of the appropriate formula and finding the necessary values to input. Details of the simplification are not dealt with in the video. Students will need practice, or otherwise be adept at handling the simplification of complicated fractional expressions with radicals.



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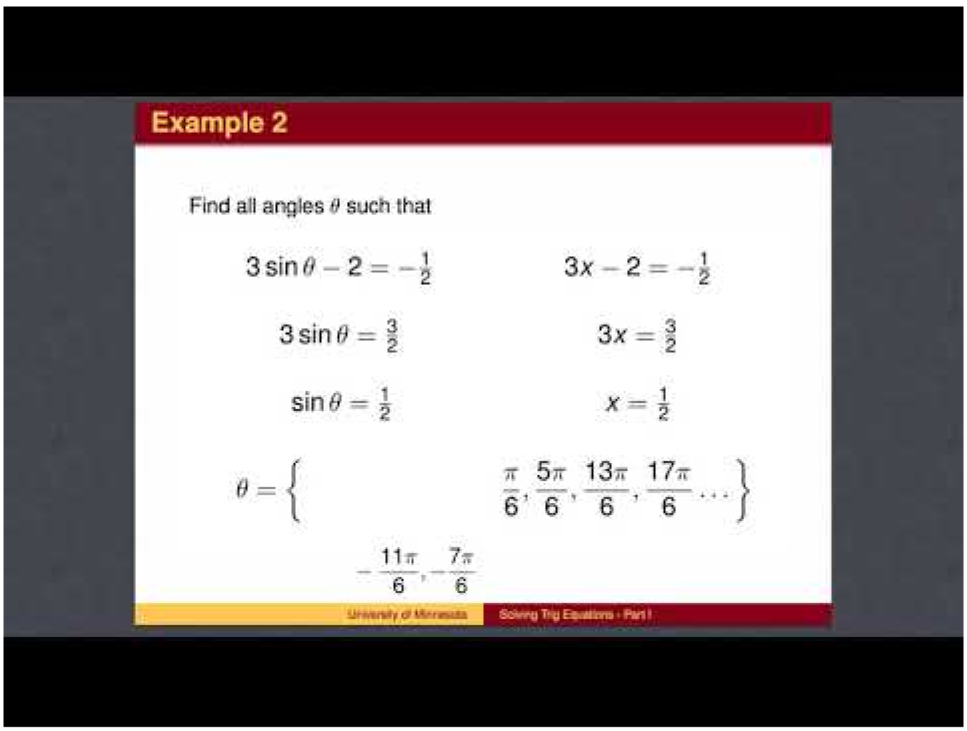
Topic 3.8
Solving Trig Equations - Part I

Solving Trig Equations – Part I covers examples where it is necessary to find a set of angles that solve an equation involving trig functions. We choose to find **all** solutions, not just those on the interval $[0, 2\pi)$ to emphasize the periodicity of the solutions. Several examples employ algebraic techniques and students may need to review solving algebraic equations. Further examples involving more complicated angles are saved for *Part II*.



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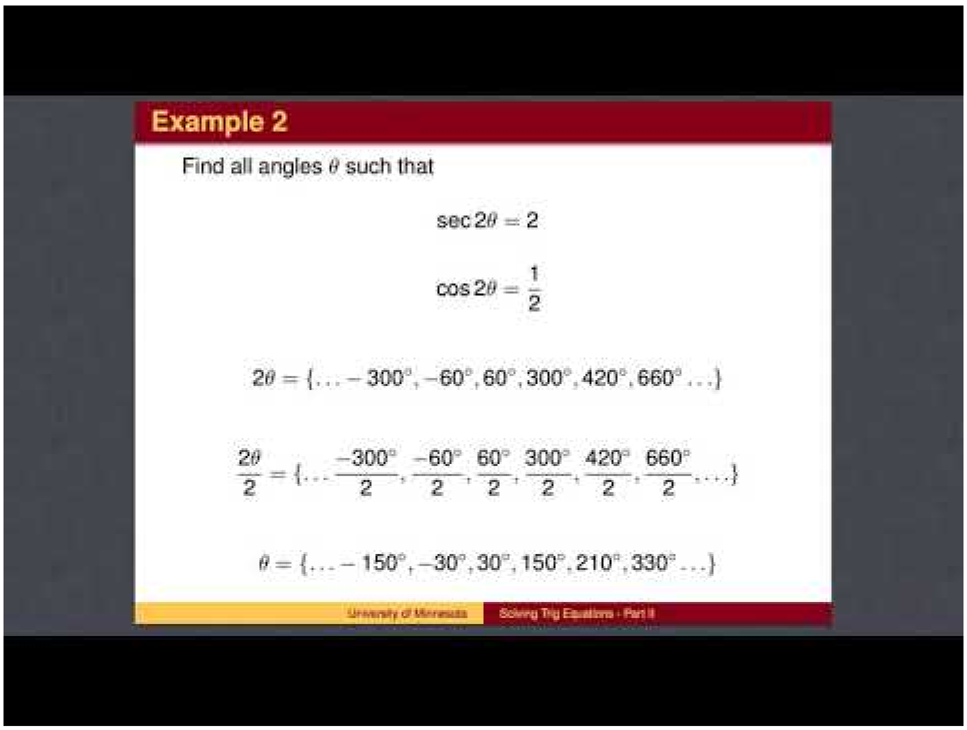
Topic 3.9
Solving Trig Equations - Part II

Solving Trig Equations – Part II covers examples where the angle is more complicated than simply θ. Students will need to perform algebraic steps, both before and after finding angles that solve a simple trig equation.



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Chapter 4 - Solving Triangles

Chapter 4 uses the Law of Sines and Law of Cosines to extend the ability to solve triangles beyond just right triangles. Formulas for the area of a triangle using trig functions can also be found.

Activities - Chapter 4

Activity 4a – Congruent Triangles

This activity explores the issues involved with solving triangles using the Law of Sines and Law of Cosines. It is a geometric exploration and ideas learned here will help in the decision-making process for solving triangles. Some familiarity with geometric proofs that triangles are congruent may be helpful, but is not necessary.

Congruent Triangles

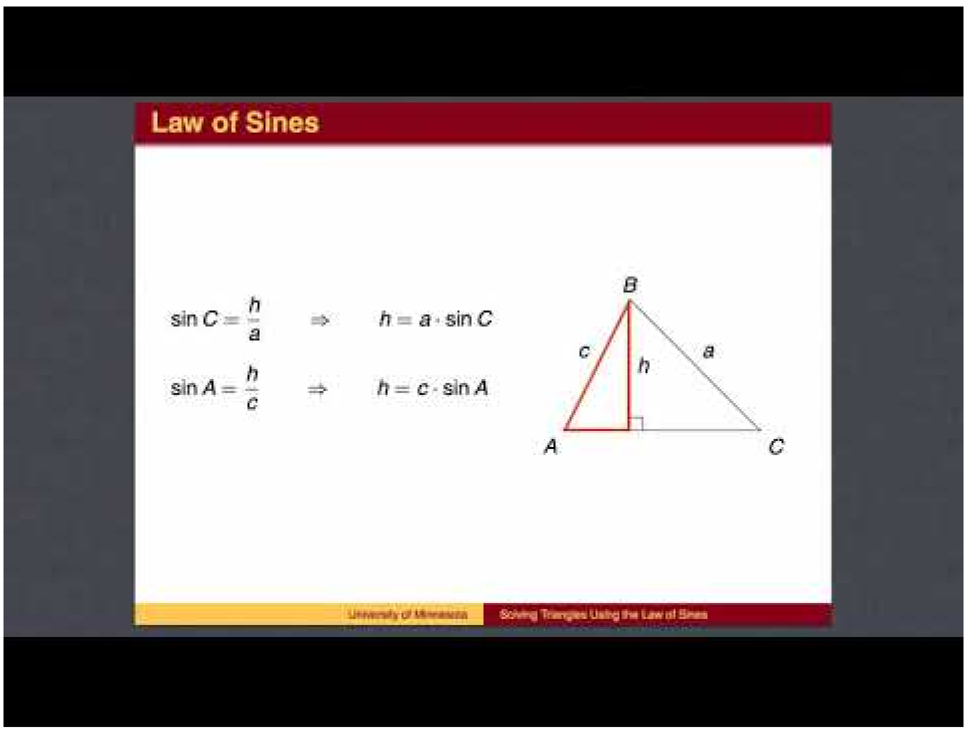
Activity 4b – Area of a Triangle

This activity develops area formulas for triangles in the SAS case and the ASA/AAS case. It uses the Law of Sines and Law of Cosines from Topics 4.1, 4.2 and 4.3. There is also an area formula in the SSS case, but it is not presented in this activity.

Area of a Triangle

Topic 4.1
Solving Triangles Using the Law of Sines - Part I

Solving Triangles Using the Law of Sines – Part I develops the Law of Sines, and uses it to solve triangles in the Angle-Side-Angle (ASA) case and the Angle-Angle-Side (AAS) case, though only the first case is shown. The Side-Angle-Side (SAS) case and the Side-Side-Side (SSS) case are shown in *Solving Triangles Using the Law of Cosines*. The ambiguous case, the Side-Side-Angle (SSA) case is shown in *Solving Triangles Using the Law of Sines – Part II*.



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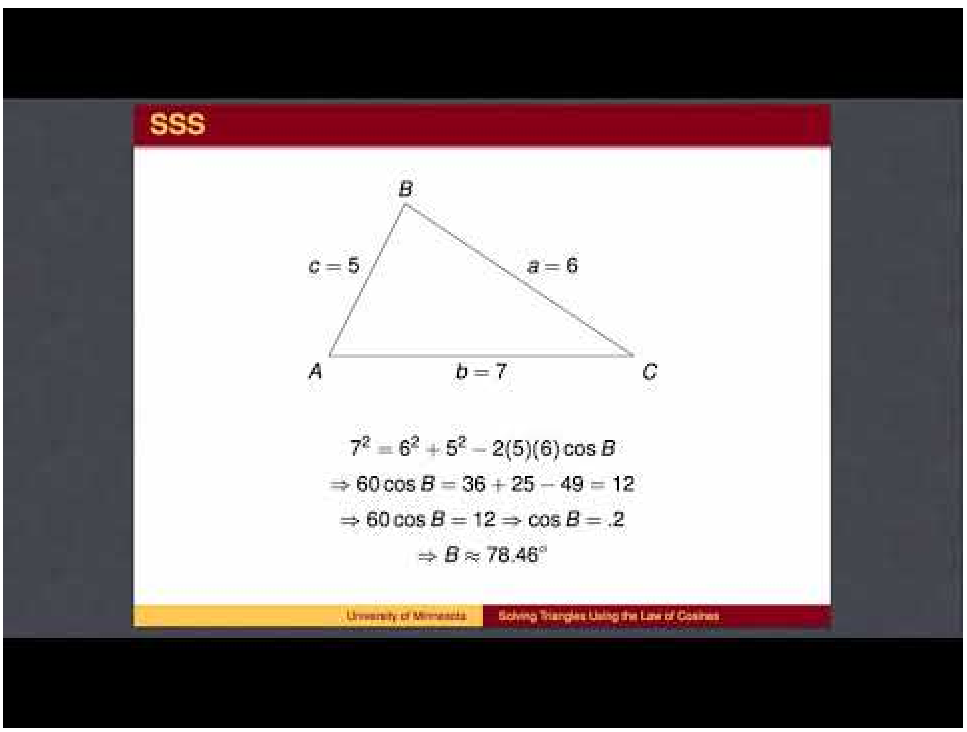
Topic 4.2
Solving Triangles Using the Law of Cosines

Solving Triangles Using the Law of Cosines solves triangles in the Side-Angle-Side (SAS) and Side-Side-Side (SSS) cases. It refers to the *Law of Sines* to complete the solving process, so familiarity with *Solving Triangles Using the Law of Sines – Part I* is helpful, but not necessary. Each problem could be solved using only the Law of Cosines. The remaining case, the ambiguous Side-Side-Angle (SSA) case is shown in *Solving Triangles Using the Law of Sines – Part II*.

The proof of the *Law of Cosines* is not given here, only examples where the *Law of Cosines* is used to solve triangles.



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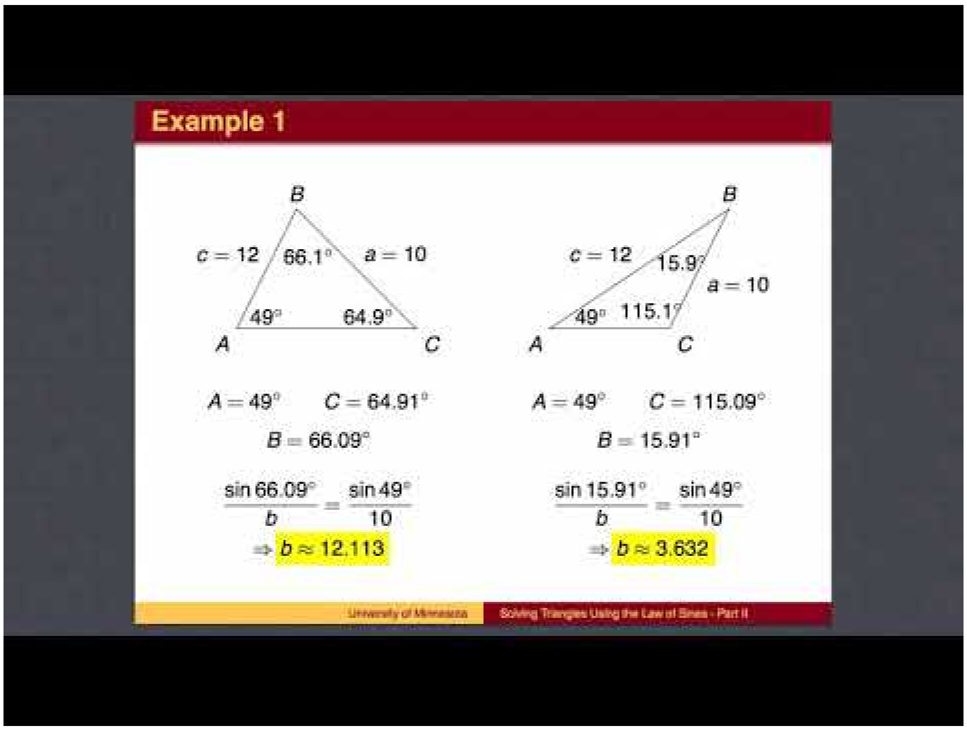
Topic 4.3
Solving Triangles Using the Law of Sines - Part II

Solving Triangles Using the Law of Sines – Part II covers the ambiguous Side-Side-Angle (SSA) case, which may produce two solutions. It relies on *Solving Triangles Using the Law of Sines – Part I*.



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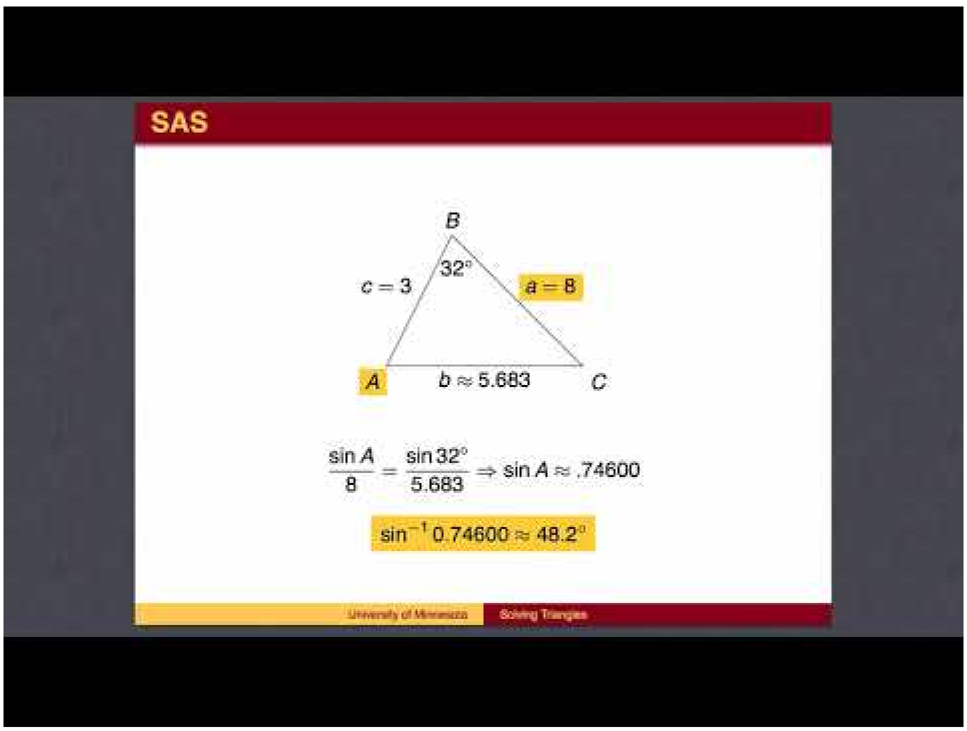
Topic 4.4
Solving Triangles

Solving Triangles summarizes how to solve triangles using the *Law of Sines* and *Law of Cosines*. The details of each case are given in the earlier lessons on *Solving Triangles Using the Law of Sines – Part I*, *Solving Triangles Using the Law of Cosines*, and *Solving Triangles Using the Law of Sines – Part II*.



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Chapter 5 - Vector Applications

Chapter 5 introduces the concepts and notation for vectors, with applications to physical forces and navigation.

Activities - Chapter 5

Activity 5a – Vector Form of the Area of a Triangle

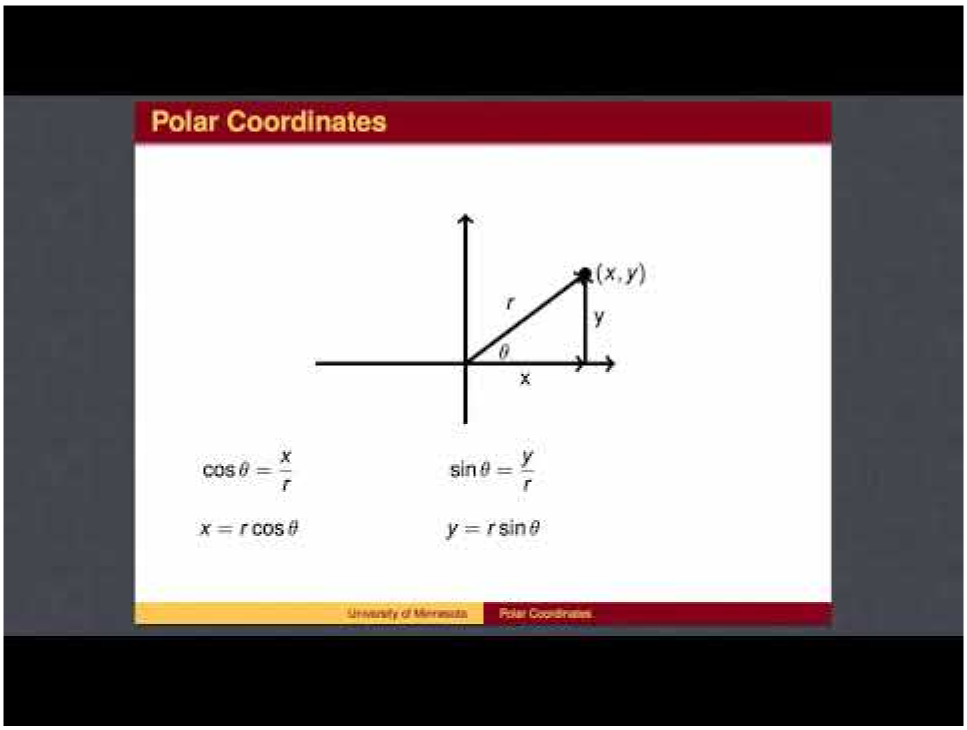
This activity develops a simple formula for the area of a triangle when two sides are given in vector form. It has applications to physics with which you may be familiar. You should be familiar with the formulas for the area of a triangle in Activity 4b. Several topics from Units 3, 4 and 5 will also appear in this activity, it is a good review of much of the material up to this point.

Vector Form of Area

Topic 5.1
Polar Coordinates

Polar Coordinates covers the navigational system using distance and direction from a given point, compared to the Cartesian 'city grid' system. Conversions between Polar Coordinates and Cartesian Coordinates are stressed. This lesson prepares students for the study of vectors and these conversion reappear in the lesson on *Applications of Vectors*.

Equations of r as a function of θ are not discussed here.



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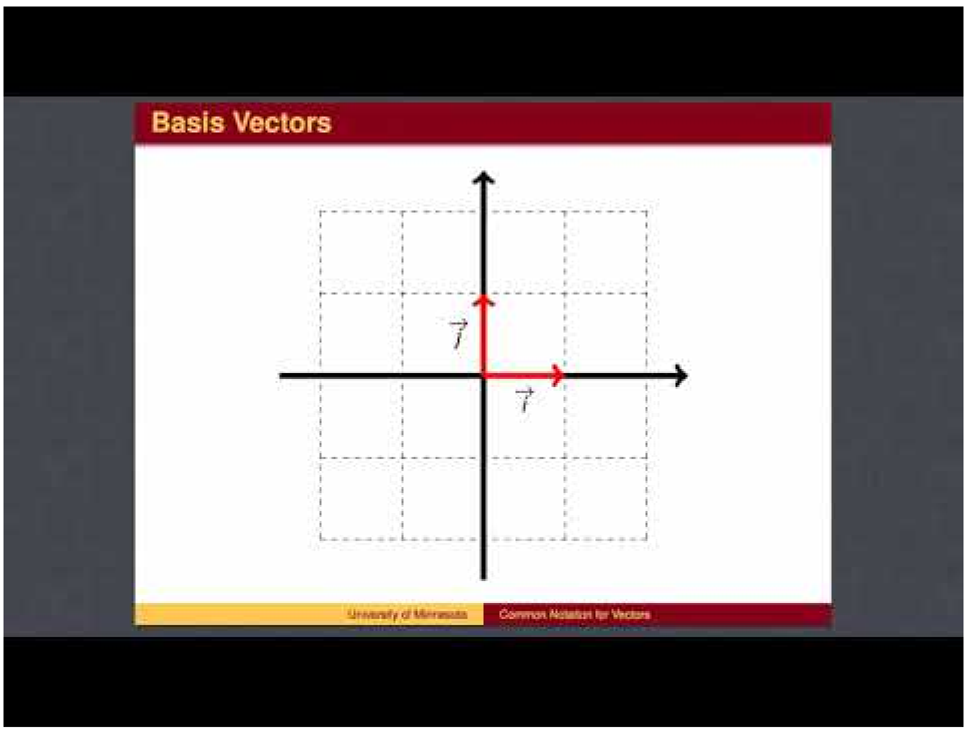
Topic 5.2
Common Notation for Vectors

Common Notation for Vectors introduces the **basis vectors** $\vec{i}$ and $\vec{j}$, along with terminology and notation used in vector applications. It reviews the rectangular to polar conversion. There is little mathematical content in this lesson, the focus instead being on familiarity with the presentation seen in vector applications within other disciplines.



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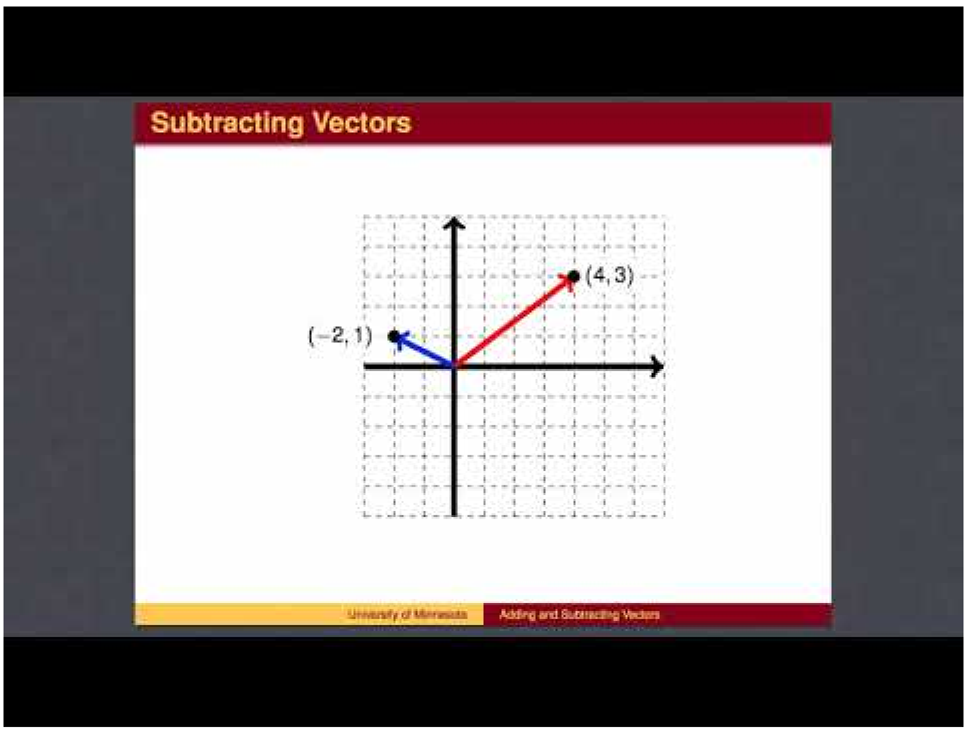
Topic 5.3
Adding and Subtracting Vectors

Adding and Subtracting Vectors develops the standard operations (addition, scalar multiplication) for two-dimensional vectors. The graphic interpretation is shown, as well as the formal computations. This lesson is the final step in the preparation for *Applications of Vectors*.



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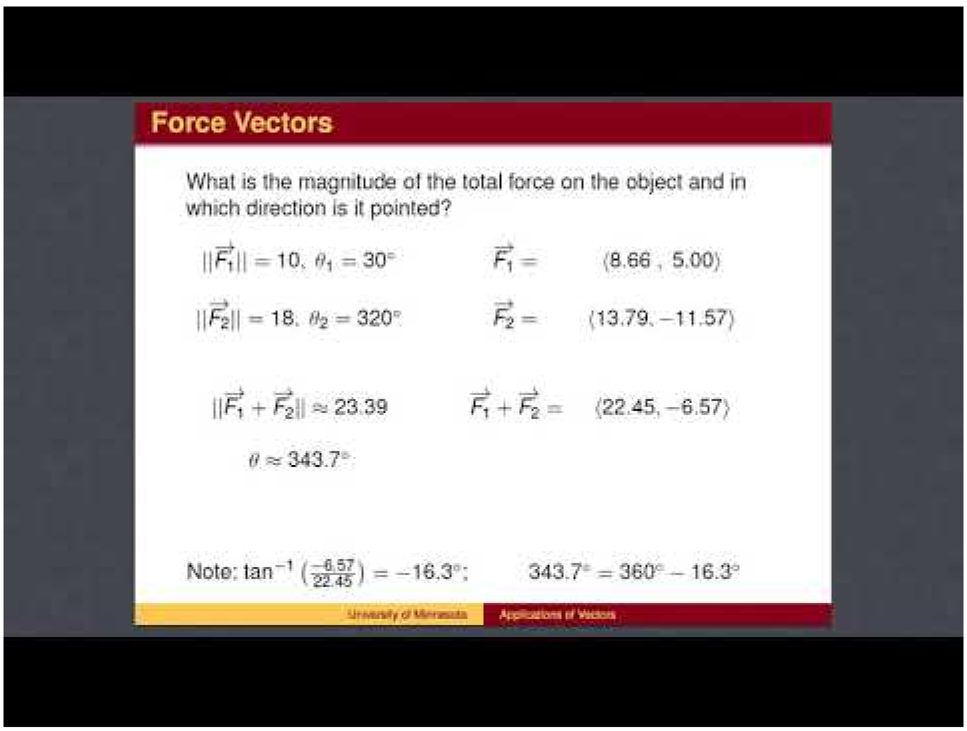
Topic 5.4
Applications of Vectors

Applications of Vectors covers two examples. The first is adding physical force vectors, the second is the navigation of a ship in moving water (a current) which is equivalent to plane flying in moving air (wind). The initial vectors are given in polar form and need to be added by converting to Cartesian coordinates.



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Chapter 6 - Conic Sections

Chapter 6 covers conic sections (circle, ellipse, parabola, hyperbola) including completing the square to convert from general form to standard form. Rotations are not discussed, as they are more appropriate for a course in linear algebra.

Activities - Chapter 6

Activity 6a – Completing the Square

This activity is designed to develop the 'Completing the Square' technique. You should focus on the technique itself, and not the results. The idea is that many graphs rely on a standard form which give a lot of information about the graph.

Examples include:

- Equation of a Circle: $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the center of the circle and r is the radius.
- Equation of a Parabola: $y=a(x-h)^2+k$, where (h,k) is the vertex, and a determines a vertical stretch factor.
- Equation of an Ellipse: $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$, where (h,k) is the center, and a and b determine the lengths of the major and minor axes.
- Equation of a Hyperbola: $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$, where (h,k) is the center, and a and b determine the slopes of the asymptotes.

The common theme is that the variables x and y only appear in terms that are squared. There are no linear terms like $3x$ or $-8y$. The key, therefore, is to rewrite equations so that terms involving x and y can be factored as perfect squares. This is where the name of the technique comes from, rearrange terms to find constants to be added that form a pattern that can be factored; these constants `complete' the square.

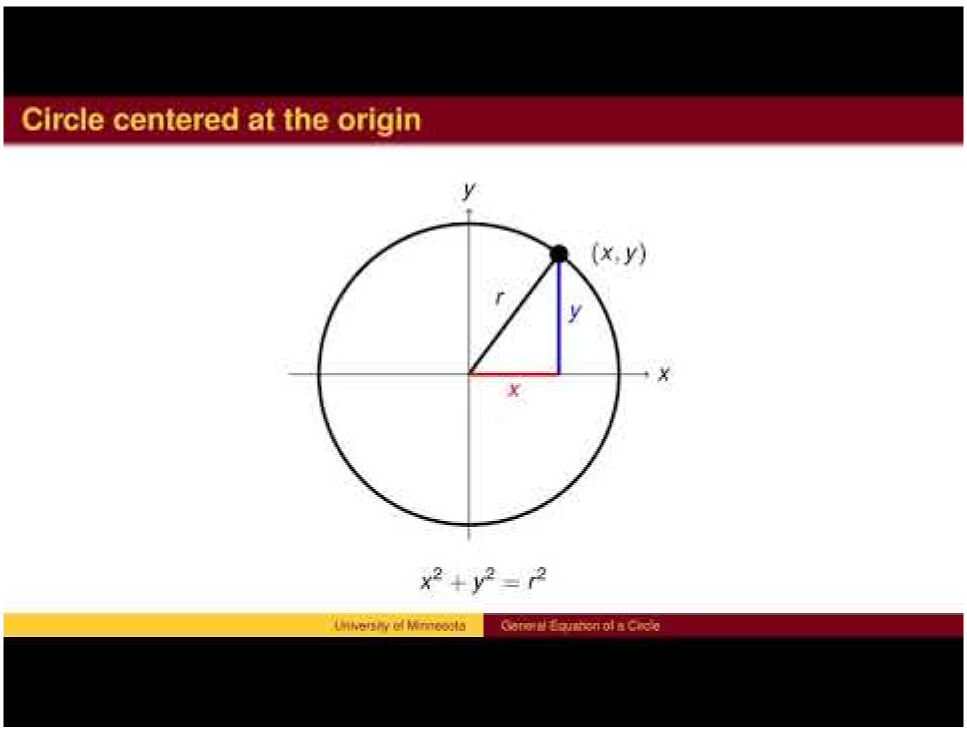
Students should focus on the goal equation, which factors will occur, and which constants will produce these factors. The process often goes in reverse, starting with the proposed factors, and working backward to find the constants. Students are best served if they understand the factors, and think about factoring issues, rather than trying to figure out the constants initially.

This activity will enable students to convert the general form of conic sections (circle, ellipse, parabola, hyperbola) to the standard form listed above.

Completing the Square

Topic 6.1
General Equation of a Circle

The *General Equation of a Circle* builds the equation from the Pythagorean Theorem. Of fundamental importance is the geometric definition and the method by which we convert geometric descriptions into algebraic formulas. The lesson on *Analytic Geometry* may be useful for students trying to understand the connection between algebra and geometry. The conversion from the general form to the standard form is saved for a lesson on *Completing the Square*.



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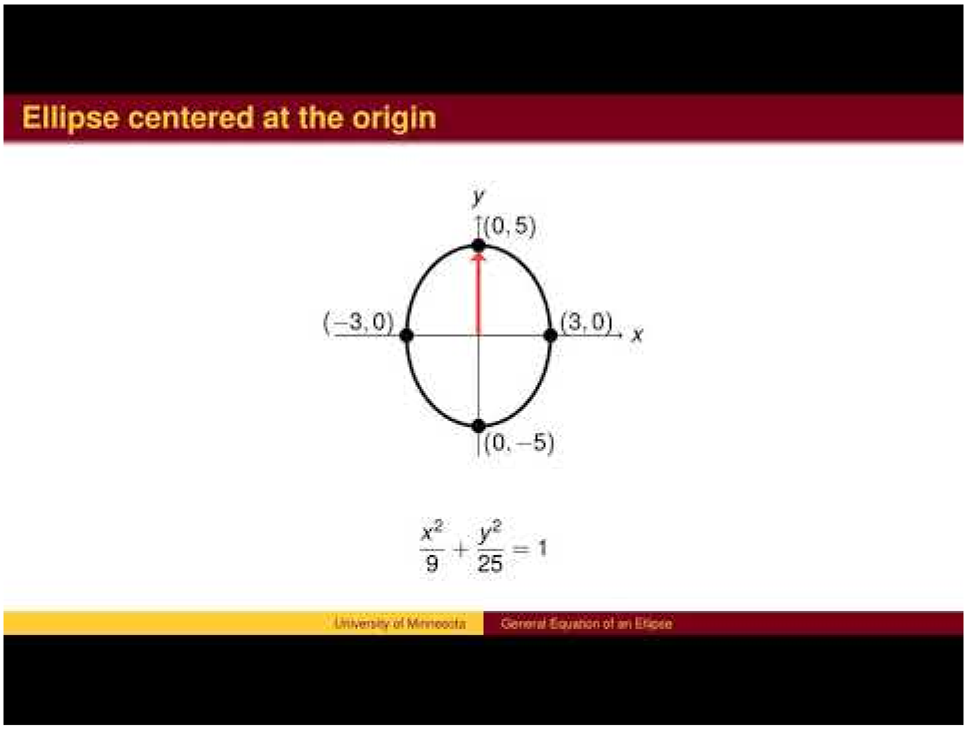
Topic 6.2
General Equation of an Ellipse

The *General Equation of an Ellipse* expands on the *General Equation of a Circle* by applying graph transformations to stretch the axes. The conversion from the general form to the standard form is saved for a lesson on *Completing the Square*.



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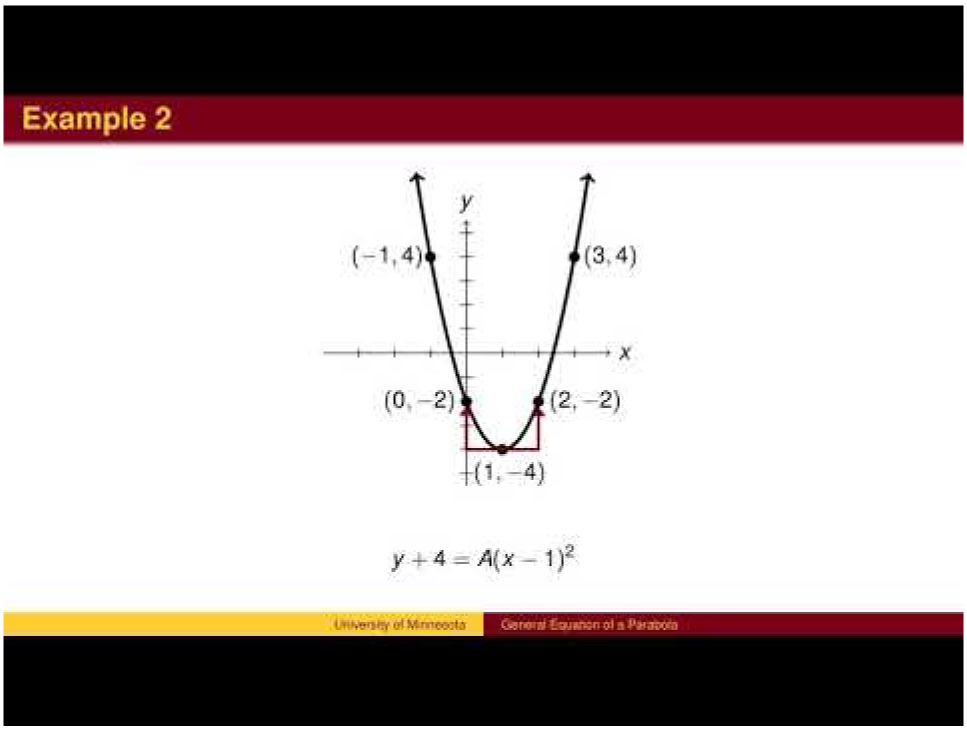
Topic 6.3
General Equation of a Parabola

The *General Equation of a Parabola* applies graph transformations to the standard parabola $y = x^2$. It also discusses the parabola $x = y^2$. The conversion from the general form to the standard form is saved for a lesson on *Completing the Square*.



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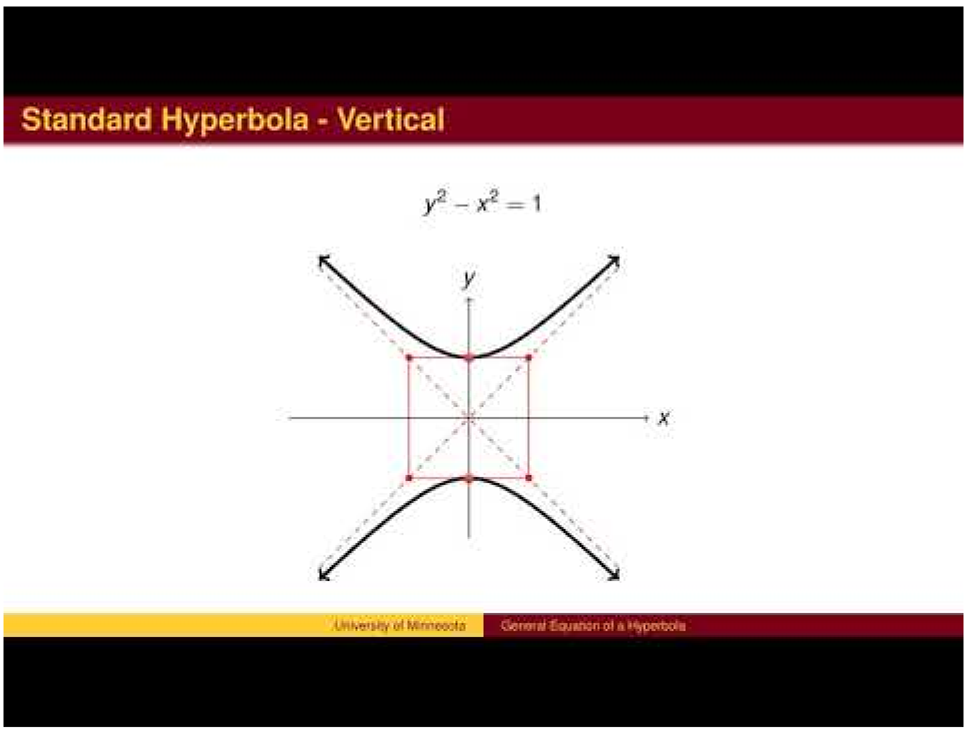
Topic 6.4
General Equation of a Hyperbola

The *General Equation of a Hyperbola* continues the basic graphing of conic sections and the equations which represent them. The lesson relies on the students' knowledge of graph transformations. The conversion from the general form to the standard form is saved for a lesson on *Completing the Square*.



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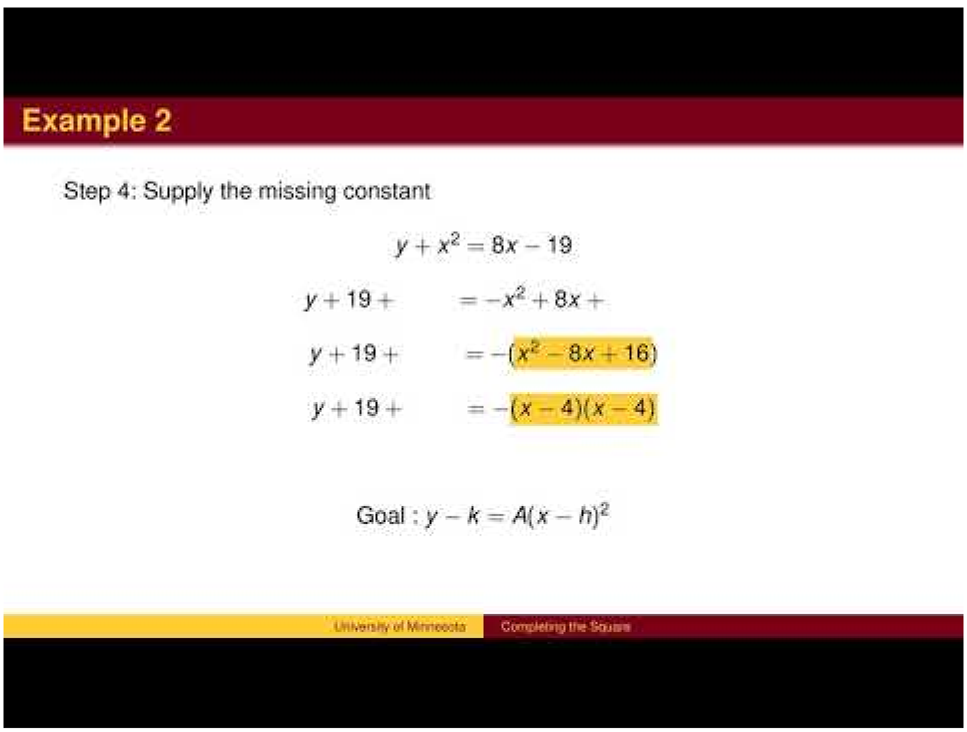
Topic 6.5
Completing the Square

Completing the Square covers the technique to convert the general form of a conic section to standard form. The process here does not involve any memorization, instead focusing on the final form from the beginning, and retroactively supplying missing constants from the desired factors. Three examples are shown (circle, parabola, hyperbola). Knowledge of conic sections is reviewed early in the lesson.



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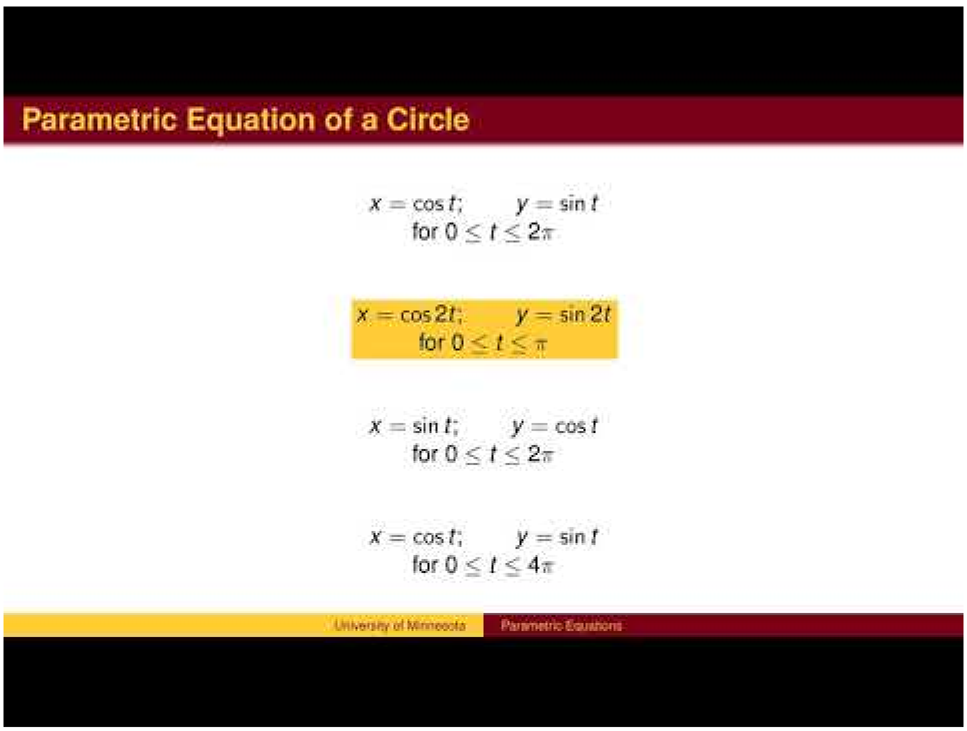
Topic 6.6
Parametric Equations

Parametric Equations discusses the basic framework for describing a particle in the xy-plane moving through time by defining the horizontal and vertical components separately by the functions $x(t)$ and $y(t)$. The examples of a line segment and an ellipse are shown in detail.



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